Multimodal authentication: impact of blind dimensionality reduction

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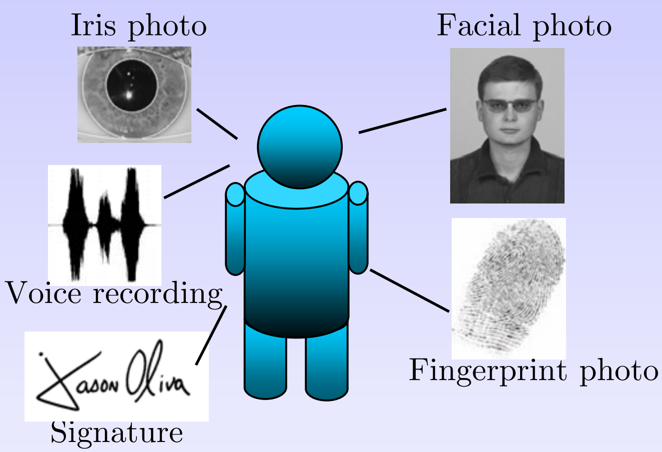
Agenda

- Introduction
- Generic one-vs-one testing problem
- Optimal multimodal dimensionality reduction
- Blind multimodal dimensionality reduction
- Conclusions and research perspectives

Introduction: Multimodal objects

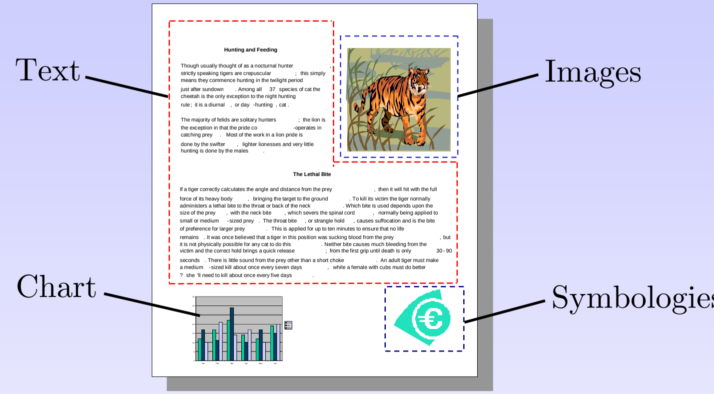
Multimodal objects

Humans



Biometrics

Documents/Slides/Presentations



Modalities

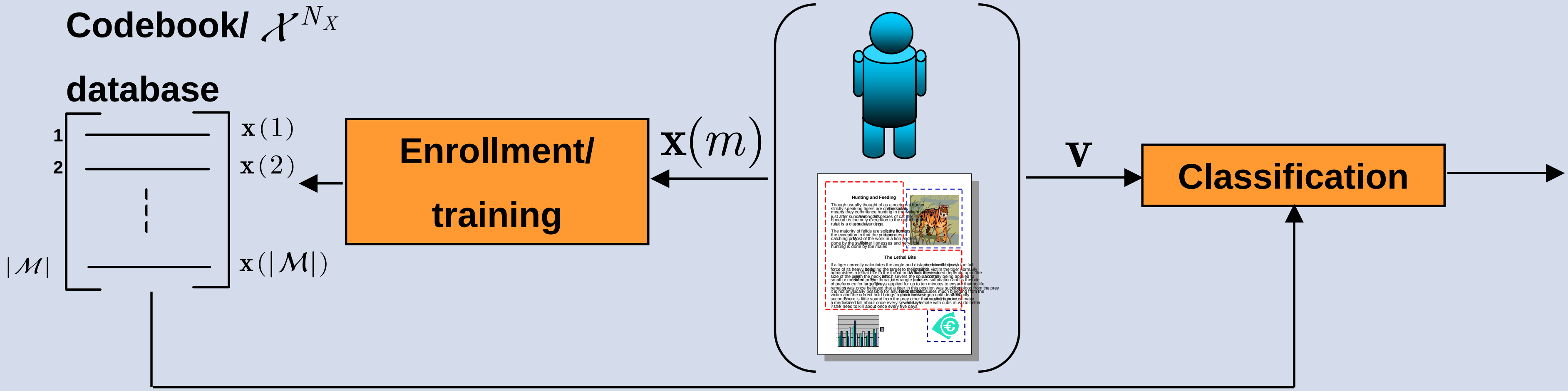
- **Images**
- **Drawings/Charts**
- **Text**
- **Symbologies**

Ex.: CHASM database, EPFL biometric recognition

Introduction: Main setups in multimodal fusion

Most of multimodal fusion problems in information retrieval, indexing, person/document/event classification and interaction, etc. can be reduced to two basic hypothesis testing problems:

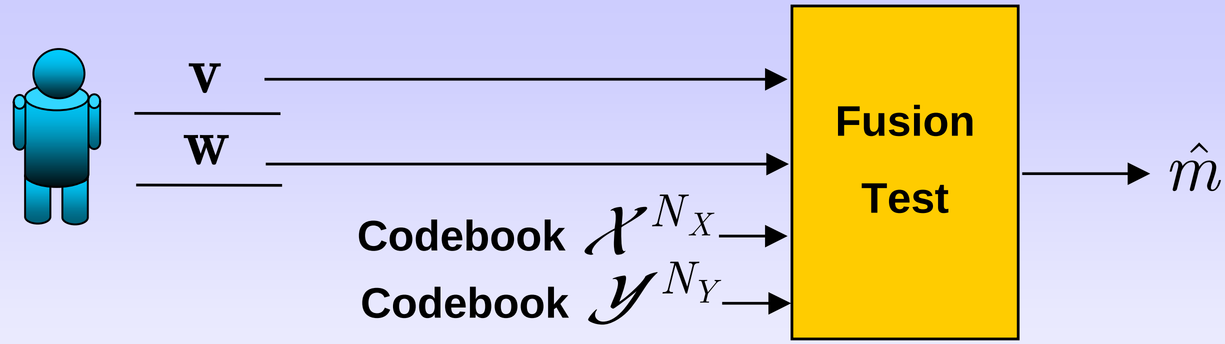
- **one-vs-one testing** also known as **authentication**;
- **one-vs-many testing** also known as **identification**.



Introduction: Main setups in multimodal fusion

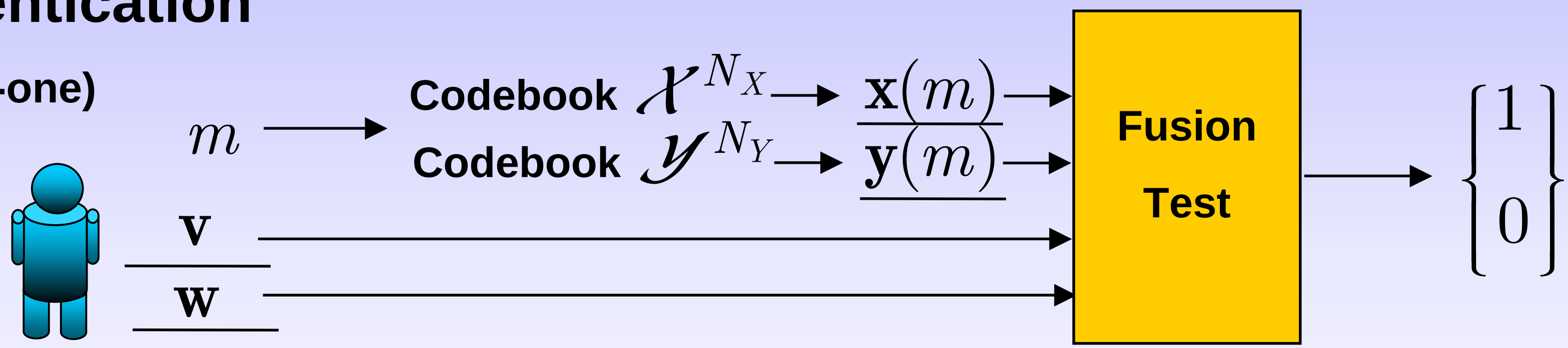
Identification

(one-vs-many)



Authentication

(one-vs-one)



Introduction: Main setups in multimodal fusion

Main common concerns of multimodal fusion:

- **optimal modality fusion depending on the dependence between modalities;**
- **high dimensionality of multimodal signals that impacts:**
  - **complexity;**
  - **storage;**
  - **priors (training sets/learning procedures).**

Solution:

Dimensionality reduction of multimodal signals related to the optimal feature extraction.

Introduction: Main setups in multimodal fusion

Dimensionality reduction techniques

highly rely on the prior knowledge of underlying statistics of:

- **modalities and their relationship;**
- **modality acquisition conditions, i.e., different distortions including noise, blur, different sampling rates, compression, desynchronisation…**

Question:

- **What can be done if the above knowledge is partial, not reliable or no priors are available at all?**

Consequence:

- **What is the loss in performance with respect to perfectly informed setup?**

Introduction: Main setups in multimodal fusion

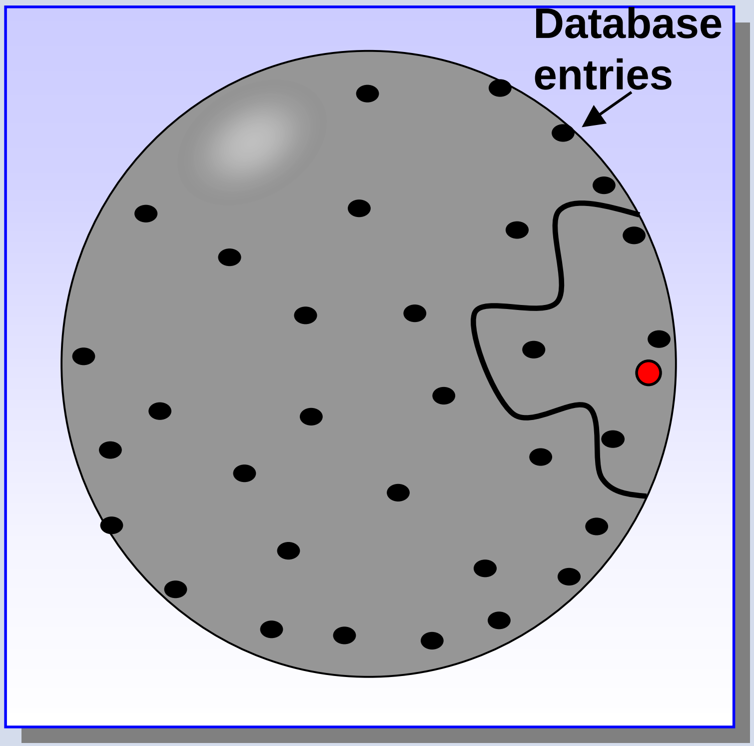
Dimensionality reduction techniques

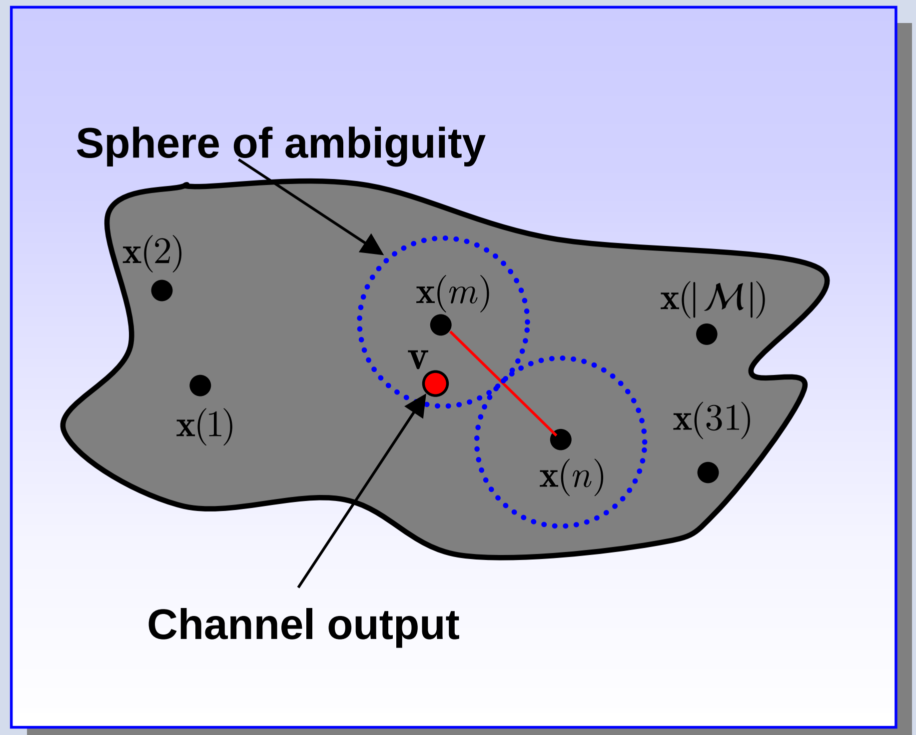
Our goal is twofold:

- **To investigate the performance of authentication under optimal and "blind" dimensionality reduction;**
- **To develop and practically demonstrate the efficiency of optimal fusion rules for the reduced dimensions.**

Generic one-vs-one testing problem

One-vs-one testing: unimodal vector space (warm up example)





$$\mathbf{X} \sim \mathcal{N}\left(0, \sigma_X^2 \mathbf{I}_N\right), \quad N \to \infty$$

Test: $\left\| \mathbf{x}(m) - \mathbf{v} \right\|^2 < e(m)$

Generic one-vs-one testing problem

One-vs-one testing: hypothesis testing (warm up example)

Authentic $$H_1 : \mathbf{V} \sim p\left(\mathbf{v} \middle| H_1\right) = p\left(\mathbf{v} \middle| \mathrm{x}(m)\right)$$

Not Authentic $$H_0 : \mathbf{V} \sim p\left(\mathbf{v} \middle| H_0\right) = \begin{cases} p\left(\mathbf{v} \middle| \mathrm{x}(1)\right), with\ probability\ p_1 \\ p\left(\mathbf{v} \middle| \mathrm{x}(2)\right), with\ probability\ p_2 \\ \vdots \\ p\left(\mathbf{v} \middle| \mathrm{x}(n)\right), with\ probability\ p_n \\ \vdots \\ p\left(\mathbf{v} \middle| \mathbf{x}(|\mathcal{M}|)\right), with\ probability\ p_{|\mathcal{M}|} \end{cases}, n \neq m$$

Generic one-vs-one testing problem: Composite Hypothesis Testing

One-vs-one testing: hypothesis testing (warm up example)

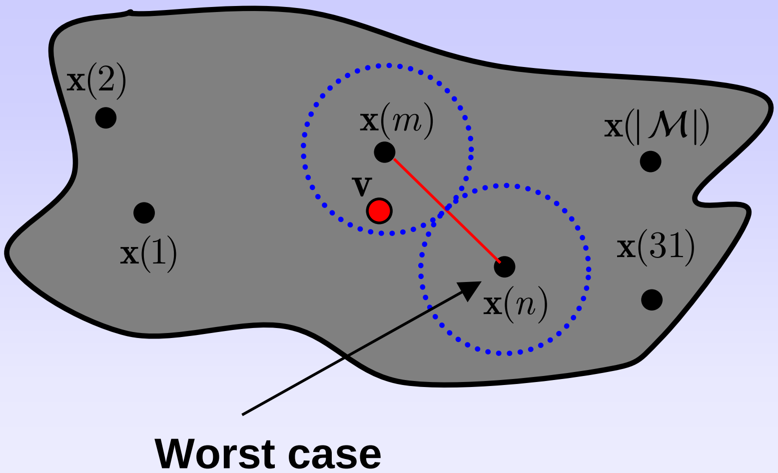
Concerns of Composite Hypothesis Testing

- **Does not coincide with the worst case setup**
- **Cumbersome integration for the composite hypothesis** H_0
- **Not easy to derive closed-form results**

Alternative Approach:

- **Consider only the worst-case setup**

One-vs-one testing: unimodal vector space (warm up example)



For m**, select:** $\mathbf{V} \sim p(\mathbf{v}|\mathbf{x}(n))$

$$\begin{cases} H_1 : \mathbf{V} \sim p(\mathbf{v}|H_1) = p(\mathbf{v}|\mathrm{x}(m)), & \textbf{(Authentic)} \\ H_0 : \mathbf{V} \sim p(\mathbf{v}|H_0) = p(\mathbf{v}|\mathrm{x}(n)). & \textbf{(Not Authentic)} \end{cases}$$

Generic one-vs-one testing problem: Test

One-vs-one testing: multimodal formulation

Hypothesis

$$\begin{cases} H_1 : (\mathbf{V},\mathbf{W}) \sim p(\mathbf{v},\mathbf{w}|H_1) = p(\mathbf{v}|H_1)\,p(\mathbf{w}|H_1), & \textbf{(Authentic)} \\ H_0 : (\mathbf{V},\mathbf{W}) \sim p(\mathbf{v},\mathbf{w}|H_0) = \underbrace{p(\mathbf{v}|H_0)\,p(\mathbf{w}|H_0)}_{\text{independent modalities}}. & \textbf{(Not Authentic)} \end{cases}$$

Strategy: Fix P_F and minimize P_M using Neyman-Pearson test:

$$\boxed{\Lambda(\mathbf{v},\mathbf{w}) = \frac{p(\mathbf{v},\mathbf{w}|H_1)}{p(\mathbf{v},\mathbf{w}|H_0)} = \frac{p(\mathbf{v}|H_1)\,p(\mathbf{w}|H_1)}{p(\mathbf{v}|H_0)\,p(\mathbf{w}|H_0)} > \eta}$$

Generic one-vs-one testing problem: Performance

One-vs-one testing: multimodal formulation

Generic bounds on performance (do not depend on pdf and dimensionality):

ROC

$$\boxed{\begin{aligned} P_F &\le e^{\mu(s) - s\dot{\mu}(s)}, \\ P_M &\le e^{\mu(s) + (1-s)\dot{\mu}(s)}. \end{aligned}} \qquad \begin{aligned} &0 \le s \le 1 \\ &\eta = \dot{\mu}(s) \end{aligned}$$

Average probability of error

$$\boxed{P_e \le \frac{1}{2} e^{\mu(s_m)}} \qquad \begin{aligned} &s_m, \\ &s.t. \, \dot{\mu}(s) = 0 \end{aligned}$$

One-vs-one testing: multimodal formulation

Define (Chernoff distance):

$$D_s\left(p(\mathbf{v},\mathbf{w}|H_1),p(\mathbf{v},\mathbf{w}|H_0)\right)=-\log\int_{\mathcal{V}^{N_X}}\int_{\mathcal{W}^{N_Y}}p(\mathbf{v},\mathbf{w}|H_1)\left(\frac{p(\mathbf{v},\mathbf{w}|H_1)}{p(\mathbf{v},\mathbf{w}|H_0)}\right)^s d\mathbf{v}\,d\mathbf{w}$$

Define (to simplify notations)

$$\mu(s)=-D_s\left(p(\mathbf{v},\mathbf{w}|H_1),p(\mathbf{v},\mathbf{w}|H_0)\right)$$

$$\dot{\mu}(s)=-\dot{D}_s\left(p(\mathbf{v},\mathbf{w}|H_1),p(\mathbf{v},\mathbf{w}|H_0)\right)$$

Generic one-vs-one testing problem: Performance

One-vs-one testing: multimodal formulation

Additivity property of Chernoff distance (independent modalities):

$$D_s\left(p(\mathbf{v},\mathbf{w}|H_1),p(\mathbf{v},\mathbf{w}|H_0)\right)=$$
$$D_s\left(p(\mathbf{v}|H_1),p(\mathbf{v}|H_0)\right)+D_s\left(p(\mathbf{w}|H_1),p(\mathbf{w}|H_0)\right)$$

Note: the same property is valid for the other IT distances including KLD (for the proof use chain rule and positivity property).

Conclusion: combination of independent modalities indeed increases overall Chernoff distance and thus reduces all types of errors!

Generic one-vs-one testing problem: Performance

One-vs-one testing: multimodal formulation

Worst case setup:

- **minimum possible distance between the distributions corresponding to two hypothesis:**
  - **General case: minimum Chernoff distance among all distributions in the database;**
  - **Gaussian case: minimum Euclidian distance among all codewords;**
- **the largest sphere of ambiguity corresponding to the acquisition conditions:**
  - **it is known that the Gaussian has the largest entropy (ambiguity) among all distributions with the bounded variance.**

Generic one-vs-one testing problem: Performance

One-vs-one testing: multimodal formulation

Worst case setup

$$\begin{cases} H_1 : \begin{cases} \mathbf{v} = \mathrm{x}(m) + \mathbf{z}_X, \\ \mathbf{w} = \mathbf{y}(m) + \mathbf{z}_Y, \end{cases} \\ H_0 : \begin{cases} \mathbf{v} = \mathrm{x}(n) + \mathbf{z}_X, \\ \mathbf{w} = \mathbf{y}(n) + \mathbf{z}_Y. \end{cases} \end{cases}$$

Assumptions:

- **modalities can have any distributions but we consider the minimum distance case for each modality,**
- **noise in each modality:** $\mathbf{Z}_X \sim \mathcal{N}\left(0, \sigma^2_{Z_X}\mathbf{I}_{N_X}\right), \mathbf{Z}_Y \sim \mathcal{N}\left(0, \sigma^2_{Z_Y}\mathbf{I}_{N_Y}\right)$

Generic one-vs-one testing problem: Performance

One-vs-one testing: multimodal formulation

Hypothesis

$$\begin{cases} H_1 : (\mathbf{V}, \mathbf{W}) \sim p(\mathbf{v}, \mathbf{w} | H_1) = \mathcal{N}\left(\mathbf{x}(m), \sigma^2_{Z_X} \mathbf{I}_{N_X}\right) \mathcal{N}\left(\mathbf{y}(m), \sigma^2_{Z_Y} \mathbf{I}_{N_Y}\right), \\ H_0 : (\mathbf{V}, \mathbf{W}) \sim p(\mathbf{v}, \mathbf{w} | H_0) = \mathcal{N}\left(\mathbf{x}(n), \sigma^2_{Z_X} \mathbf{I}_{N_X}\right) \mathcal{N}\left(\mathbf{y}(n), \sigma^2_{Z_Y} \mathbf{I}_{N_Y}\right). \end{cases}$$

Define: the worst case distances

$$d_X^2 = \|\mathbf{x}(m) - \mathbf{x}(n)\|^2$$

$$d_Y^2 = \|\mathbf{y}(m) - \mathbf{y}(n)\|^2$$

Generic one-vs-one testing problem: Performance

One-vs-one testing: multimodal formulation

Performance

$$P_D(P_F) = Q\left(Q^{-1}(P_F) - \sqrt{\frac{d_X^2}{\sigma_{Z_X}^2} + \frac{d_Y^2}{\sigma_{Z_Y}^2}}\right)$$

$$P_e = Q\left(\frac{1}{2}\sqrt{\frac{d_X^2}{\sigma_{Z_X}^2} + \frac{d_Y^2}{\sigma_{Z_Y}^2}}\right)$$

Conclusion:

- **performance depends on ratio of worst case distance to noise variance;**
- **presence of additional modality with any non-zero worst-case distance leads to performance enhancement, if fusion is performed optimally.**

One-vs-one testing: optimal dimensionality reduction

Dimensionality reduction

$$\boxed{\begin{aligned} \tilde{\mathbf{x}} &= \Phi_{\mathbf{x}}\mathbf{x}, \\ \tilde{\mathbf{y}} &= \Phi_{\mathbf{y}}\mathbf{y}. \end{aligned}}$$

$$\Phi_{\mathbf{x}} \in \mathbb{R}^{L_X \times N_X}, \Phi_{\mathbf{y}} \in \mathbb{R}^{L_Y \times N_Y}$$

$$\mathbf{x} \in \mathbb{R}^{N_X}, \mathbf{y} \in \mathbb{R}^{N_Y}$$

$$\tilde{\mathbf{x}} \in \mathbb{R}^{L_X}, \tilde{\mathbf{y}} \in \mathbb{R}^{L_Y}$$

What for:

- **reduction of complexity;**
- **memory storage;**
- **easier classifier training, design and performance analysis.**

Optimal multimodal dimensionality reduction

One-vs-one testing: optimal dimensionality reduction

Dimensionality reduction

$$\boxed{\begin{aligned} \tilde{\mathbf{x}} &= \Phi_{\mathbf{x}}\mathbf{x}, \\ \tilde{\mathbf{y}} &= \Phi_{\mathbf{y}}\mathbf{y}. \end{aligned}}$$

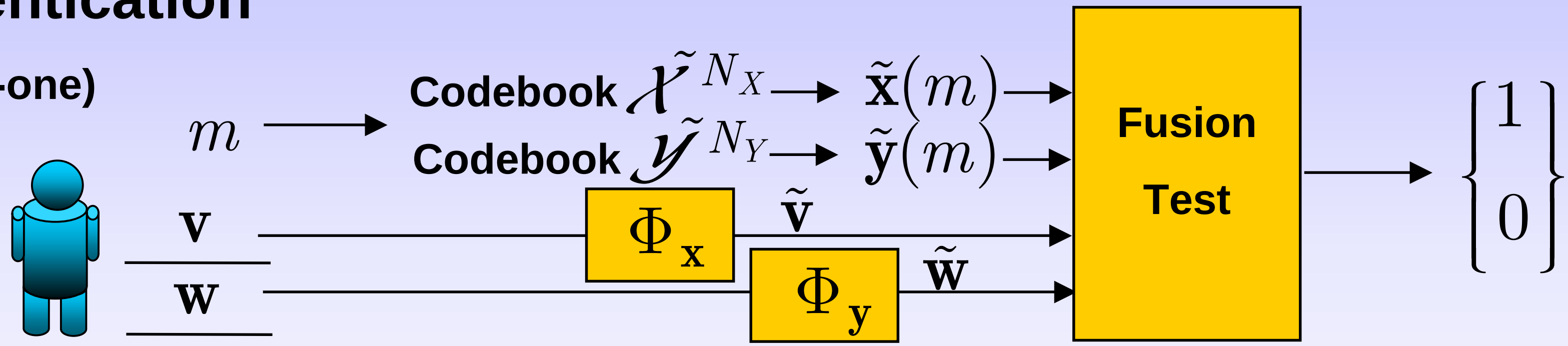
$$\Phi_{\mathbf{x}} \in \mathbb{R}^{L_X \times N_X}, \Phi_{\mathbf{y}} \in \mathbb{R}^{L_Y \times N_Y}$$

$$\mathbf{x} \in \mathbb{R}^{N_X}, \mathbf{y} \in \mathbb{R}^{N_Y}$$

$$\tilde{\mathbf{x}} \in \mathbb{R}^{L_X}, \tilde{\mathbf{y}} \in \mathbb{R}^{L_Y}$$

Authentication

(one-vs-one)



Optimal multimodal dimensionality reduction

One-vs-one testing: optimal dimensionality reduction

State-of-the-art:

There exist many linear and nonlinear dimensionality reduction techniques.

Main strategy:

Design transform that maximizes/minimizes some objective criterion using all available priors (or training data).

Our approach: objective criterion is

$$\boxed{\left(\Phi_{\mathbf{x}},\Phi_{\mathbf{y}}\right)=\arg\max_{\Phi_{\mathbf{x}},\Phi_{\mathbf{y}}} D_s\left(p(\tilde{\mathbf{v}},\tilde{\mathbf{w}}|H_1),p(\tilde{\mathbf{v}},\tilde{\mathbf{w}}|H_0);\Phi_{\mathbf{x}},\Phi_{\mathbf{y}}\right)}$$

Maximization of Chernoff distance will lead to the minimization of errors.

One-vs-one testing: optimal dimensionality reduction

Solution:

$$\Phi_{\mathbf{x}}^{opt} = (\mathbf{x}(m) - \mathbf{x}(n))^T \frac{1}{\sigma^2_{Z_X}},$$

$$\Phi_{\mathbf{y}}^{opt} = (\mathbf{y}(m) - \mathbf{y}(n))^T \frac{1}{\sigma^2_{Z_Y}}.$$

Conclusion: **optimal dimensionality reduction transform requires:**

- **knowledge of worst-case vectors among all in advance;**
- **knowledge of worst-case variances for each modality;**
- **addition of new entry requires to redesign the optimal transform.**

One-vs-one testing: blind dimensionality reduction

Open issue: **it is difficult to cope with the above issues in practice.**

Solution: "blind" dimensionality reduction.

Strategy behind: what can be achieved, if to relax the constraint on the minimum dimensionality but applying completely "blind" dimensionality reduction technique?

One possible approach: random projections in the class of orthoprojectors, i.e.: $\Phi_{\mathbf{x}}\Phi_{\mathbf{x}}^T = \mathbf{I}_{L_X}, \Phi_{\mathbf{y}}\Phi_{\mathbf{y}}^T = \mathbf{I}_{L_Y}$

Blind multimodal dimensionality reduction

One-vs-one testing: blind dimensionality reduction

Main concerns:

- **how to apply to different modalities?**
- **what is the impact on performance as a function of** $\frac{L_X}{N_X}$ **and** $\frac{L_Y}{N_Y}$ **?**

Blind multimodal dimensionality reduction

One-vs-one testing: blind dimensionality reduction

Hypothesis

$$\begin{cases} H_1 : \left(\tilde{\mathbf{V}}, \tilde{\mathbf{W}}\right) \sim p(\tilde{\mathbf{v}}, \tilde{\mathbf{w}} | H_1) = \mathcal{N}\left(\tilde{\mathbf{x}}(m), \sigma^2_{Z_X} \Phi_{\mathbf{x}} \Phi_{\mathbf{x}}^T\right) \mathcal{N}\left(\tilde{\mathbf{y}}(m), \sigma^2_{Z_Y} \Phi_{\mathbf{y}} \Phi_{\mathbf{y}}^T\right), \\ H_0 : \left(\tilde{\mathbf{V}}, \tilde{\mathbf{W}}\right) \sim p(\tilde{\mathbf{v}}, \tilde{\mathbf{w}} | H_0) = \mathcal{N}\left(\tilde{\mathbf{x}}(n), \sigma^2_{Z_X} \Phi_{\mathbf{x}} \Phi_{\mathbf{x}}^T\right) \mathcal{N}\left(\tilde{\mathbf{y}}(n), \sigma^2_{Z_Y} \Phi_{\mathbf{y}} \Phi_{\mathbf{y}}^T\right). \end{cases}$$

Define: the worst case distances in the random projection domain

$$\tilde{d}_X^2 = \left(\mathbf{x}(m) - \mathbf{x}(n)\right)^T \Phi_{\mathbf{x}}^T \left(\Phi_{\mathbf{x}} \Phi_{\mathbf{x}}^T\right)^{-1} \Phi_{\mathbf{x}} \left(\mathbf{x}(m) - \mathbf{x}(n)\right)$$

$$\tilde{d}_Y^2 = \left(\mathbf{y}(m) - \mathbf{y}(n)\right)^T \Phi_{\mathbf{y}}^T \left(\Phi_{\mathbf{y}} \Phi_{\mathbf{y}}^T\right)^{-1} \Phi_{\mathbf{y}} \left(\mathbf{y}(m) - \mathbf{y}(n)\right)$$

Blind multimodal dimensionality reduction

One-vs-one testing: blind dimensionality reduction

Performance

$$P_D(P_F) = Q\left(Q^{-1}(P_F) - \sqrt{\frac{\tilde{d}_X^2}{\sigma_{Z_X}^2} + \frac{\tilde{d}_Y^2}{\sigma_{Z_Y}^2}} \right)$$

$$P_e = Q\left(\frac{1}{2} \sqrt{\frac{\tilde{d}_X^2}{\sigma_{Z_X}^2} + \frac{\tilde{d}_Y^2}{\sigma_{Z_Y}^2}} \right)$$

Conclusion:

- **performance depends on ratio of worst case distance to noise variance;**
- **presence of additional modality with any non-zero worst-case distance again leads to performance enhancement, if fusion is performed optimally.**

Blind multimodal dimensionality reduction

One-vs-one testing: blind dimensionality reduction

Approximation: Consequence of the Johnson-Lindenstrauss Lemma:

$$(1-\xi)\sqrt{\frac{L}{N}} < \frac{\|\Phi\mathbf{x}\|}{\|\mathbf{x}\|} < (1+\xi)\sqrt{\frac{L}{N}}$$

$$0 < \xi < 1$$

Using this result, we can approximate the effect of the projection:

$$(1-\xi)\sqrt{\frac{L}{N}}\,\|\mathbf{x}\| < \|\Phi\mathbf{x}\| < (1+\xi)\sqrt{\frac{L}{N}}\,\|\mathbf{x}\|$$

Blind multimodal dimensionality reduction

One-vs-one testing: blind dimensionality reduction

Performance

$$P_D(P_F) \approx Q\left(Q^{-1}(P_F) - \sqrt{\frac{L_X}{N_X}\frac{d_X^2}{\sigma_{Z_X}^2} + \frac{L_Y}{N_Y}\frac{d_Y^2}{\sigma_{Z_Y}^2}} \right)$$

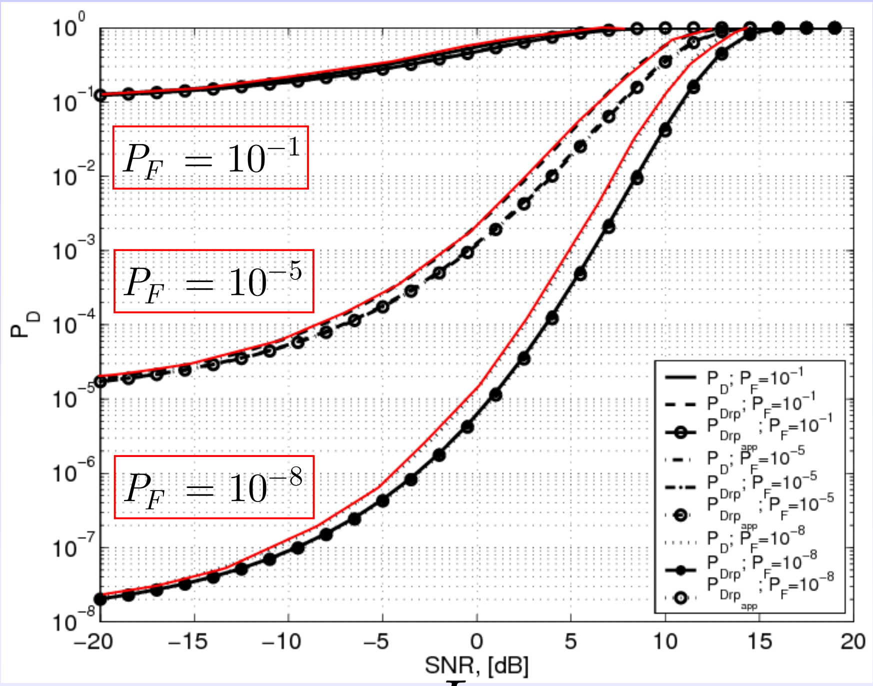
$$P_e \approx Q\left(\frac{1}{2}\sqrt{\frac{L_X}{N_X}\frac{d_X^2}{\sigma_{Z_X}^2} + \frac{L_Y}{N_Y}\frac{d_Y^2}{\sigma_{Z_Y}^2}} \right)$$

Conclusion:

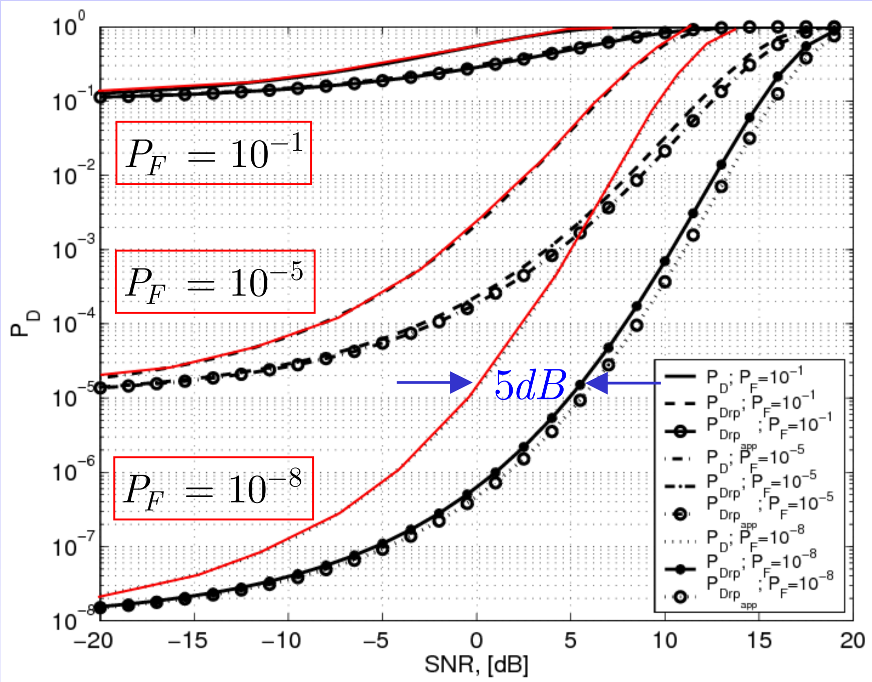
- **performance loss is proportional to $\frac{L_X}{N_X}$ and $\frac{L_Y}{N_Y}$ in each modality!**

Blind multimodal dimensionality reduction: Unimodal case

Probability of correct detection P_D **(ROC)** $SNR = 10\log_{10}\frac{\|\mathbf{x}\|^2}{\sigma_Z^2}$



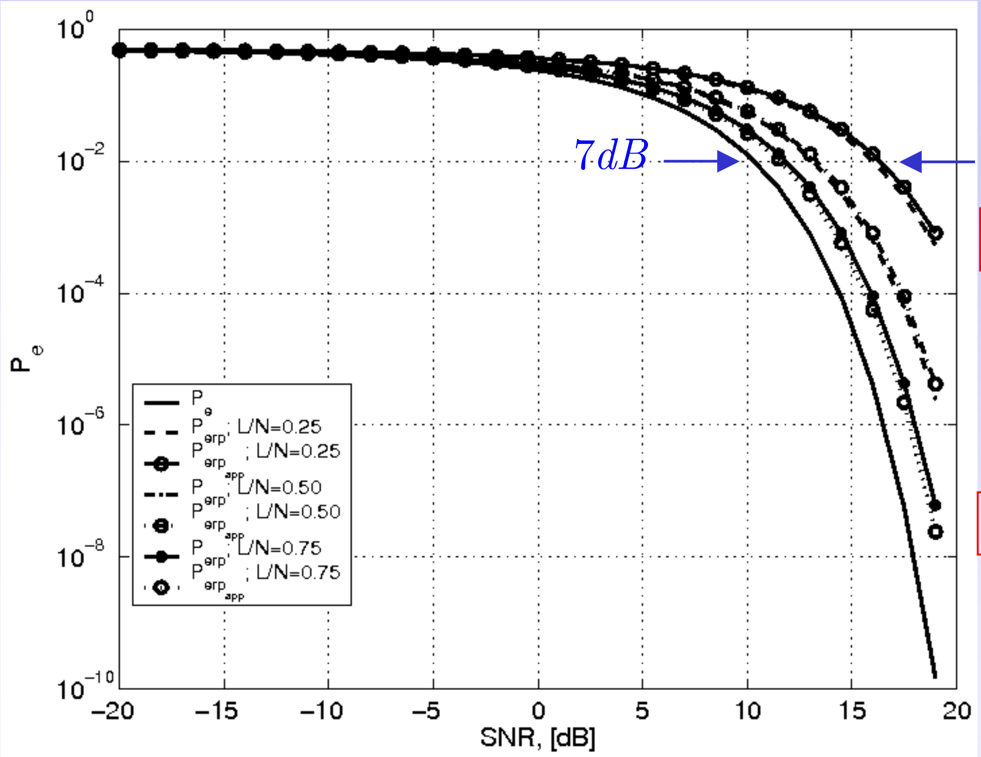
$$\frac{L}{N} = 0.75$$



$$\frac{L}{N} = 0.25$$

Blind multimodal dimensionality reduction: Unimodal case

Average probability of error P_e



$L/N = 0.25$

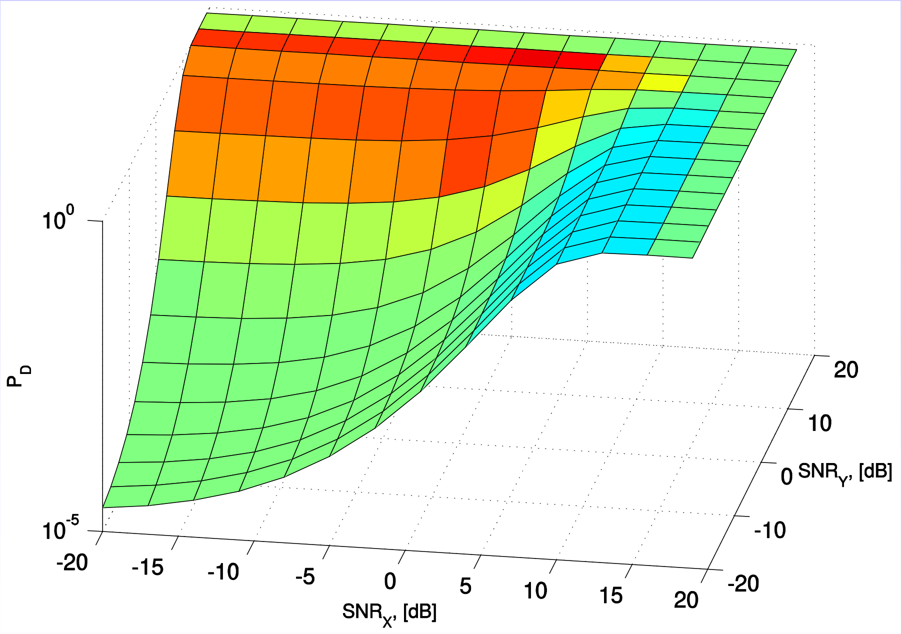
$L/N = 0.5$

$L/N = 0.75$

No Projection

Blind multimodal dimensionality reduction: Multimodal case

Probability of correct detection P_D



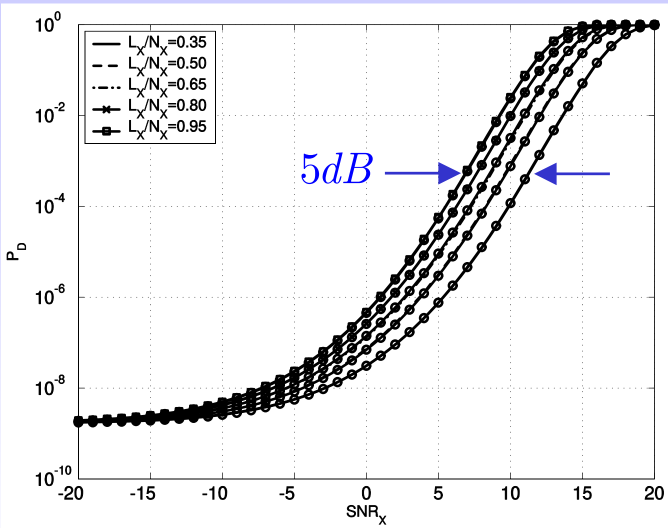
$$P_F = 10^{-5}$$

$$\frac{L_X}{N_X} = \frac{L_Y}{N_Y} = 1$$

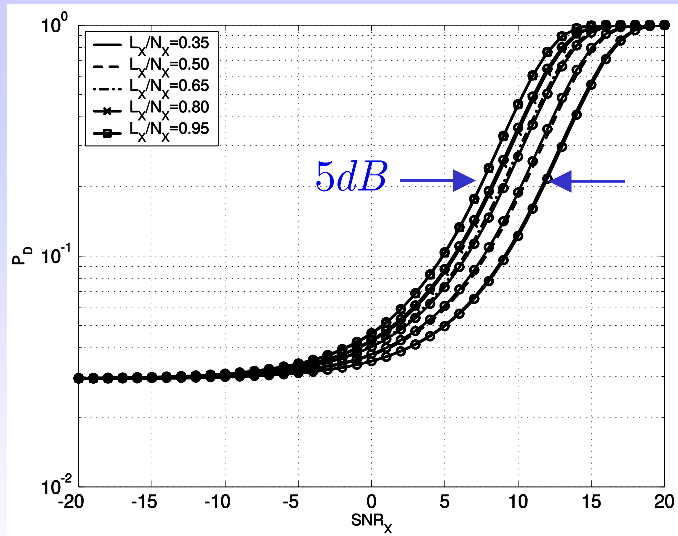
The second modality (even noisy) always enhances performance!

Blind multimodal dimensionality reduction: Multimodal case

Probability of correct detection P_D



$SNR_Y = -10dB$



$SNR_Y = +10dB$

$$P_F = 10^{-10}$$

$$\frac{L_Y}{N_Y} = 1$$

The second modality (even noisy) always enhances performance!

Conclusions and research perspectives

We have investigated the impact of dimensionality reduction on the authentication performance in terms of ROC and average probability of error.

Main lessons:

- **To investigate this impact we have considered two setups:**
  - **informed setup when all multimodal and acquisition statistics are known in advance (or at least can be learned with some accuracy from training data);**
  - **"blind" setup when the above statistics are assumed to be unknown.**
- **The dimensionality reduction can be performed independently for each modality without loss in performance for each modality (good message for complexity!).**
- **The class of optimal projectors in the blind setup is quite broad but we have focused on the <u>random projections</u> (subclass of orthoprojectors) for the simplicity of theoretical analysis and approximations.**

Conclusions and research perspectives

Main lessons:

Why is it important?

- **We know the optimal dimensionality reduction technique, which <u>does not produce any loss</u> in performance with respect to the raw data based authentication in the informed setup.**
- **However, if the statistics are unknown or difficult to learn we propose to use the blind dimensionality reduction based on random orthoprojectors, which under the worst conditions produces <u>only about 5-7dB loss</u> wrt the informed setup.**

Conclusions and research perspectives

Main lessons:

Why is it important?

- **The addition of new entries to the database requires to update the optimal dimensionality reduction each time for the informed setup!**

Is it practical? Of cause, NOT.

Contrarily, the blind dimensionality reduction based on random projections can be performed without taking into account these new entries!

This is a sort of universal feature extraction known in the information theory for source and channel coding under prior ambiguity.

Conclusions and research perspectives

Main lessons:

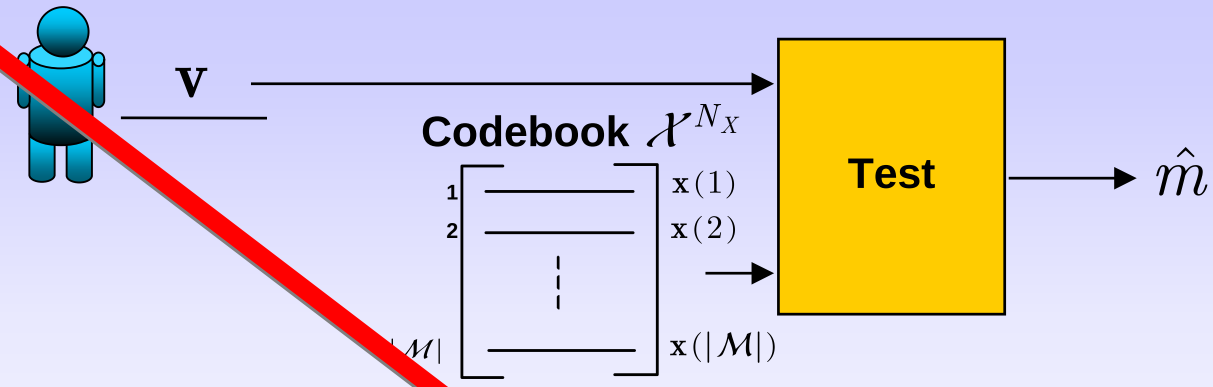
Why is it important?

- **We can use quite accurate approximations based on the J-L lemma to analytically establish the impact of dimensionality reduction based on orthoprojectors.**
- **We analytically established that this loss is proportional to the squared root of dimensionality reduction ratio, .i.e., $\sqrt{L/N}$, that is a very useful bound for the multimodal fusion in the reduced dimensionality space.**
- **The performed simulations confirmed that the approximation is very accurate for the case of two modalities.**

Basic classification problems: warm up (unimodal formulation)

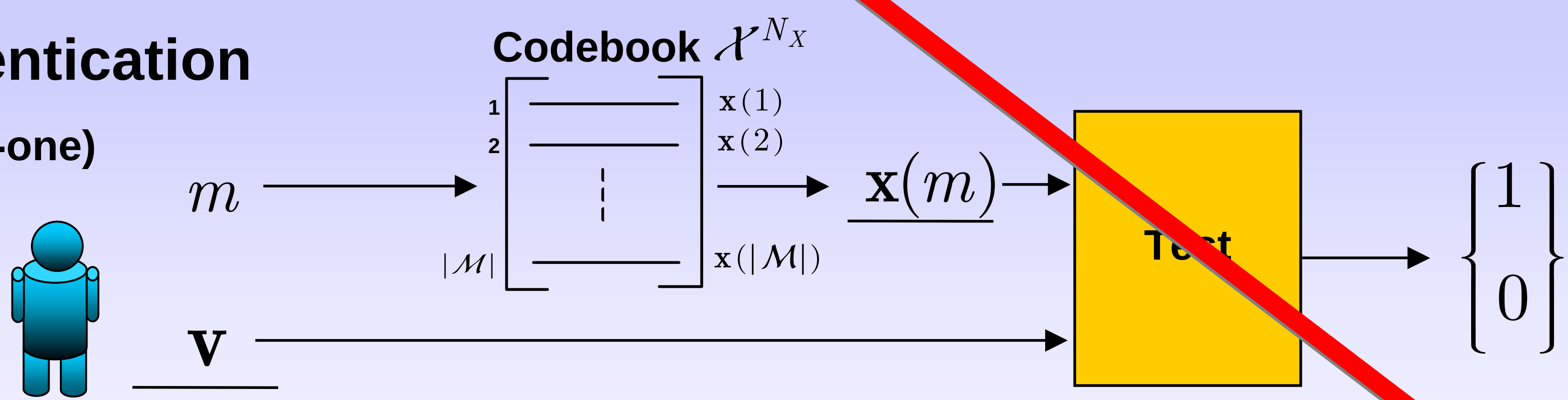
Identification

(one-vs-many)



Authentication

(one-vs-one)



Multimodal fusion: problem 2

One-vs-one testing: unimodal vector space (warm up example)

Evaluation of performance

Receiver operational characteristic (ROC)

$$P_M = \Pr[H_0 | H_1]$$ **Probability of miss**

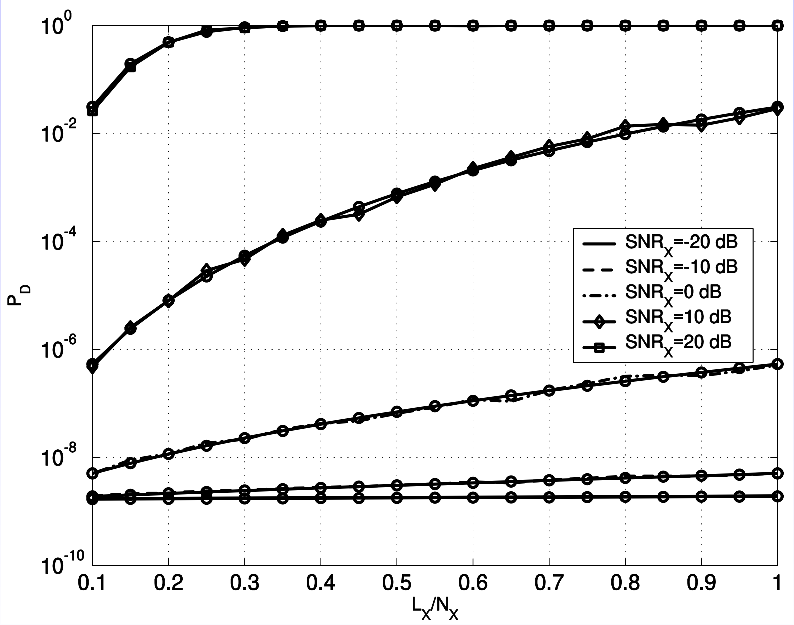
$$P_F = \Pr[H_1 | H_0]$$ **Probability of false alarm**

$$P_D = 1 - P_M$$ **Probability of correct detection**

Average probability of error

$$P_e = \frac{1}{2} P_F + \frac{1}{2} P_M$$ **Average probability of error**

Probability of correct detection P_D



$$P_F = 10^{-10}$$

$$\frac{L_Y}{N_Y} = 1$$

$$SNR_Y = -10dB$$

Main lessons

Part 1: We have investigated the impact of additional modality on the authentication performance in terms of ROC and average probability of error.

Main lessons:

- **To avoid information loss we considered the authentication based on optimal raw data fusion (it is important to have upper achievable results according to data processing inequality).**
- **The presence of additional modality (even highly noisy) always enhances performance in terms of both ROC and average probability of error for any generic assumption behind the underlying hypothesis. It is shown for:**
  - **any modality distributions based on Chernoff distances and bounds;**
  - **worst case Gaussian distribution of acquisition and worst case intramodality distances;**
  - **even independent modalities!**

Main lessons

Part 1: We have investigated the impact of additional modality on the authentication performance in terms of ROC and average probability of error.

Main lessons:

Why is it important?

- **We now know optimal fusion rules that do not produce any loss of information!**
- **We have analytical formulas that allow to optimally select modalities to achieve the best performance and thus to compromise complexity for a given application.**
- **We know the most favourable and the worst conditions for acquisition and intramodal features (minimum pairwise Chernoff distance condition);**
- **The analytical results are confirmed by computer simulation for several different modalitity distributions, dimensionalities and cardinalities of databases as well as acquisition conditions.**

Research perspectives

Part 3: We would like to extend the considered methodology to the identification setup.

Part 4: We would like to establish the link with the robust hashing as an optimal dimensionality reduction technique providing the easiest search, indexing, retrieval, identification and authentication in the large databases.

Part 5: It would be interesting to link the results with the fast search techniques in the reference list space as another alternative technique recently discovered in the identification applications.

Part 6: It would be interesting to test the developed methodology on various multimodal setups. All suggestions are highly welcome!

Dimensionality reduction techniques

Problem formulation:

Consider multimodal one-vs-one (authentication) problem on the example of human multibiometrics.

Authentication

(one-vs-one)